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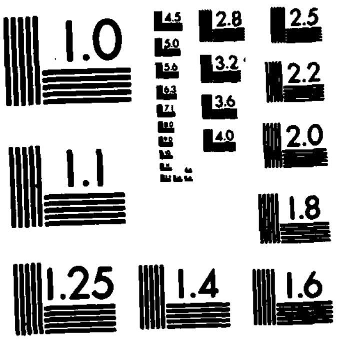

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by

D. B. Owen and Youn-Min Chou

Technical Report No. 173
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DEPARTMENT OF STATISTICS

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PREDICTION INTERVALS USING EXCEEDANCES FOR AN ADDITIONAL THIRD STAGE SAMPLE

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Key Words: Normal Prediction Intervals; Sample Size Tables; Factors for Three-Stage Sampling

Reader Aids -

Purpose: Widen state of the art.

Special math needed for explanations: Probability distributions

Special math needed for results: Probability distributions

Results useful to: Statistically inclined reliability engineers, statisticians

ABSTRACT

Prediction intervals are extended to a third sampling stage involving the distribution of the t-th smallest value in a third stage sample which exceeds the k-th smallest value in the second sample. Procedures and tables are given for two situations. In the first situation the usual 2-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for this problem. In the second situation we know in advance that three stages will be necessary and the factors are given for the required procedure.

1. INTRODUCTION

In [2] "warranty periods" are given on the future production of m items from an s-normal population, all m of which meet a criterion which is obtained from a preliminary sample of size n. In [3] these calculations are extended so that m-k+1 out of the m future production items are to meet the warranty criterion.

This paper extends these results to a third stage. The secondary production (the future production in [2,3]) met the criterion and we are now required to go into a tertiary stage where ℓ-t+1 out of ℓ additional items are needed to meet a criterion based on the number in the third stage that exceed an order statistic from the second stage. That is, we know that at least m-k+1 out of m items produced at the secondary stage were larger than $\bar{X} - rS$ (where $\bar{X}$ and S were based on a sample of size n at the first stage) and we are required to have ℓ-t+1 out of ℓ items in the third stage larger than the k-th smallest item out of m in the second stage. The third stage must be accomplished with an s-confidence of 1-β, i.e.,the probability is 1-β that ℓ-t+1 items manufactured at the third stage will exceed the k-th smallest item at the second stage, given the k-th smallest item at the second stage is larger than $\bar{X} - rS$.

We will solve this 3-stage problem for two situations.

Model A: The criterion of [2,3] has been applied successfully. Then it is necessary for additional production to meet the criterion that ℓ-t+1 out of ℓ items on a third stage are larger than the k-th smallest item of the second stage. Here we solve for the sample size, ℓ, given the other parameters of the problem.

Model B: We know in advance that we are going to have three stages and in this case we solve for the value of r in the criterion

$$\Pr\{Y_{(k)} < Z_{(t)} \mid \bar{X} - rS < Y_{(k)}\} = 1 - \beta \tag{1.1}$$

Assumptions:

1. All observations are from an s-normal distribution with mean μ and variance σ^2.
2. The first stage observations are $X_1, X_2, \ldots, X_n$.
3. The second stage observations are $Y_1, Y_2, \ldots, Y_m$.
4. The third stage observations are $Z_1, Z_2, \ldots, Z_\ell$.
5. All of the $n+m+\ell$ observations are s-independent of each other.

Notation list:

$\bar{X}$ is the sample mean of the first stage sample of size n.

S^2 is the sample variance of the first stage sample of size n, i.e.,

$$S^2 \equiv \sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1).$$

$Y_{(k)}$ is the k-th smallest observation from the second stage sample of size m.

$Z_{(t)}$ is the t-th smallest observation from the third stage sample of size ℓ.

$V_{(k+j)}$ is the (k+j)-th order statistic from a sample of size $m + \ell$ from an s-normal distribution with mean μ and variance σ^2.

$1-\beta$ is the s-confidence that $\ell-t+1$ items manufactured at the third stage will exceed the k-th smallest item at the second stage, given that the k-th smallest item at the second stage is larger than $\bar{X} - rS$.

2. THE 3-STAGE JOINT PROBABILITY

The joint probability can be expressed as

$$\Pr\{\bar{X} - rS < Y_{(k)} < Z_{(t)}\}$$

$$= \frac{1}{\binom{m+\ell}{\ell}} \sum_{j=0}^{t-1} \binom{j+k-1}{j}\binom{m+\ell-k-j}{m-k} \Pr\{\bar{X} - rS < V_{(k+j)}\} \qquad (2.1)$$

That is, the 3-stage joint probability can be expressed as a finite sum of weighted probabilities computed by the process given in [3].

3. SAMPLE SIZE REQUIRED FOR MODEL A

We assume that the user has conducted the two-stage process as given by Fertig and Mann [3] and that the product at the second stage has met the criterion that the k-th order statistic $Y_{(k)}$ is greater than $\bar{X} - rS$ where r is obtained from tables in [3]. Now we wish to find the sample size, ℓ, required so that $\Pr\{Z_{(t)} > Y_{(k)} \mid Y_{(k)} > \bar{X} - rS\} = 1 - \beta$.

Since it seems reasonable that the same risk be applied to the third stage as was used on the second stage, we also assume that $\Pr\{Y_{(k)} > \bar{X} - rS\} = 1 - \beta$. Then we wish to find the maximum value of ℓ so that $\Pr\{\bar{X} - rS < Y_{(k)} < Z_{(t)}\} \geq (1 - \beta)^2$.

Table 1 gives the sample size, ℓ, needed at the third stage to satisfy this inequality for $\beta = 0.10, 0.05$. A dash indicates it is impossible to attain the required value of β.

The first stage sample size, n, has little effect on the probability $\Pr\{\bar{X} - rS < Y_{(k)} < Z_{(t)}\}$. As the sample size n increases from 2 to ∞, the value of ℓ decreases by at most 3. We therefore printed in Table 1 only the values of ℓ for $n = 2$ and indicate the amount of decrease in ℓ between $n = 2$ and $n = \infty$ by placing *'s after the entry with the number of *'s indicating the amount of the decrease, i.e., * indicates ℓ decreased by one between $n = 2$ and $n = \infty$, ** indicates ℓ decreased by two between $n = 2$ and $n = \infty$, etc.

4. DETERMINATION OF THE FIRST STAGE FACTOR r FOR MODEL B

In the situation considered in this section we assume that we know we are going to have three stages before the first stage is completed. That is, we want to find a value of r similar to the value of r given by the prediction intervals as presented in [3]. This time, however, we want to find r such $\Pr\{Y_{(k)} < Z_{(t)} \mid \bar{X} - rS < Y_{(k)}\} = 1 - \beta$. Table 2 gives some representative values of r.

5. EXAMPLE

We extend the numerical example given in [3, p. 177]. In that example turbine nozzles made of cast alloy MAR-M 246 cc were to have a constant load of 25K psi. A group of 50 nozzles was available and 10 of them were randomly selected and life-tested in order to predict the failure time of the remaining 40. The sample mean and sample standard deviation of the log failure times for these 10 nozzles were 3.850 and 0.034, respectively. An s-normal distribution for the failure times was assumed. For $n = 10$, $m = 40$, $k = 5$ and $\beta = 0.05$, Fertig and Mann [3] found that $r = 2.37$. Hence a 95 percent lower prediction limit for the time of failure 5 for the remaining 40 nozzles is $\exp[3.850 - (2.37)(.034)] = 43.4$ hours.

For the problem discussed in Model A of this paper, we need additional nozzles for the third stage knowing that among the 40 nozzles of the second stage the fifth failure was after 43.4 hours. Suppose that we want to be 95% sure that the fifth failure time in a third stage sample is larger than the fifth failure time on the second stage, what is the largest sample size we can take on the third stage to meet these conditions? From Table 1 with $\beta = 0.05$, $n = 10$, $m = 40$, $k = 5 = t$, we find $\ell = 14$. Hence with a sample of size 14 there is 95% s-confidence that at least 10 out of 14 of the third stage sample will have a longer life than the fifth failure time at the second stage.

For an example using the tables of Model B, we will assume that $n = 10$, $m = \ell = 40$, $k = 2$, $t = 6$ and $\beta = .10$. From Table 2, we get $r = 1.6521$. Hence our criterion for the nozzles of Fertig and Mann's [3] example is $\exp[3.850 - (1.6521)(0.034)] = 44.4$ hours. Now there is a 90% s-confidence that the sixth failure out of 40 sampled at the third stage will be greater than the second failure at the second stage knowing that the second failure out of 40 sampled at the second stage was above 44.4 hours.

ACKNOWLEDGMENTS

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REFERENCES

[1] Benjamin Epstein. "Tables for the distribution of the number of exceedances," Annals of Mathematical Statistics, 25, 1954, 762-768.

[2] K. W. Fertig and N. R. Mann. "A new approach to the determination of exact and approximate one-sided prediction intervals for normal and lognormal distributions," Reliability and Fault Tree Analysis, Edited by R. E. Barlow, J. B. Fussell, and N. D. Singpurwalla, Society for Industrial and Applied Mathematics, 1975.

[3] K. W. Fertig and N. R. Mann. "One-sided prediction intervals for at least p out of m future observations from a normal population," Technometrics, 19, 1977, 167-177.

BIOGRAPHIES

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TABLE 1

Sample Sizes Required for Third Stage (Model A)

β	k	t	m→ 20	30	40	50	60	70	80
.10	2	2	5*	7	10*	12*	14	17*	19*
		3	10*	14	19*	24*	28*	33**	38**
		4	15*	22*	29*	36*	43**	50**	57**
		5	20*	29	39*	48*	58**	68***	77***
	3	2	3	5	6	8	10*	11	13*
		3	7*	10	13	16	20*	23*	26*
		4	10	15	20	25*	30*	35*	40*
		5	14	21*	28*	34*	41*	48**	55**
	4	2	2	4*	5	6	7	9*	10*
		3	5	8*	10	13*	15	18*	20*
		4	8	12	16*	20*	23	27*	31*
		5	11	16	22*	27*	32*	37*	43**
	5	2	2	3	4	5	6	7	8
		3	4	6	8	10	12	14	16
		4	7	10	13	16	19	22	25
		5	9	14*	18*	22*	26	31*	35*
.05	2	2	3	5	6	8	9	11	12
		3	7	10	14*	17	20	23	27*
		4	11	16	22*	27*	32*	37*	42
		5	16*	23*	30*	37*	44*	51	58
	3	2	2	3	4	5	6	7	9*
		3	5	7	10*	12	14	17*	19
		4	8	12	16*	19	23	27*	30
		5	11	17*	22*	27*	32	37	42
	4	2	2	2	3	4	5	6	6
		3	4	6	7	9	11	13	15*
		4	6	9	12	15	18	21	24*
		5	9	13	17	21	25	29	33
	5	2	-	2	3	3	4	5*	5
		3	3	5	6	8*	9	11*	12
		4	5	8	10	12	15	17	20*
		5	8	11	14	18*	21	24	28*

TABLE 2

Factors for obtaining 100(1-β)% 1-sided exceedances for containing at least ℓ-t+1 out of ℓ additional observations at a third stage given a first stage sample of size n and a second stage sample of size m(=ℓ) for which at least m-k+1 out of m observations were in the prediction interval (Model B).

k= 1, t= 4, β = .10

ℓ / n	20	30	40	50	60	70	80
2	1.3345	1.4696	1.5343	1.5723	1.5962	1.6124	1.6241
4	1.6499	1.8372	1.9425	2.0113	2.0606	2.0930	2.1275
6	1.7550	1.9596	2.0794	2.1606	2.2203	2.2667	2.3042
8	1.8024	2.0150	2.1422	2.2293	2.2952	2.3467	2.3887
10	1.8274	2.0442	2.1756	2.2671	2.3360	2.3906	2.4355
12	1.8418	2.0610	2.1951	2.2891	2.3603	2.4170	2.4633
14	1.8503	2.0713	2.2071	2.3027	2.3755	2.4337	2.4819
16	1.8565	2.0779	2.2147	2.3116	2.3855	2.4448	2.4939
18	1.8603	2.0821	2.2197	2.3174	2.3922	2.4523	2.5022
20	1.8630	2.0849	2.2230	2.3213	2.3967	2.4574	2.5079
30	1.8680	2.0894	2.2283	2.3280	2.4048	2.4671	2.5191
40	1.8686	2.0889	2.2277	2.3275	2.4048	2.4675	2.5200
50	1.8683	2.0877	2.2253	2.3253	2.4031	2.4659	2.5186
60	1.8677	2.0862	2.2243	2.3239	2.4011	2.4640	2.5167
70	1.8671	2.0850	2.2227	2.3221	2.3993	2.4621	2.5148
80	1.8665	2.0839	2.2214	2.3205	2.3976	2.4603	2.5130
90	1.8660	2.0829	2.2202	2.3192	2.3962	2.4588	2.5115
∞	1.8596	2.0719	2.2061	2.3031	2.3734	2.4399	2.4915

k= 2, t= 6, β = .10

ℓ / n	20	30	40	50	60	70	80
2	.6716	.8333	.9945	1.0573	1.0935	1.1275	1.1489
4	.9955	1.2417	1.3774	1.4654	1.5273	1.5743	1.6117
6	1.1217	1.3807	1.5238	1.6276	1.6995	1.7543	1.7990
8	1.1863	1.4517	1.6064	1.7112	1.7835	1.8436	1.8972
10	1.2244	1.4934	1.6521	1.7606	1.8413	1.9046	1.9560
12	1.2490	1.5202	1.6813	1.7924	1.8754	1.9408	1.9943
14	1.2659	1.5384	1.7012	1.8140	1.8987	1.9657	2.0206
16	1.2781	1.5513	1.7154	1.8294	1.9153	1.9835	2.0395
18	1.2872	1.5609	1.7253	1.8408	1.9276	1.9966	2.0534
20	1.2943	1.5682	1.7337	1.8493	1.9368	2.0065	2.0640
30	1.3137	1.5873	1.7539	1.8712	1.9606	2.0321	2.0915
40	1.3222	1.5948	1.7614	1.8792	1.9692	2.0414	2.1015
50	1.3268	1.5985	1.7649	1.8826	1.9728	2.0454	2.1053
60	1.3296	1.6005	1.7666	1.8843	1.9745	2.0472	2.1078
70	1.3315	1.6018	1.7675	1.8851	1.9753	2.0480	2.1086
80	1.3328	1.6026	1.7680	1.8855	1.9756	2.0483	2.1089
90	1.3338	1.6031	1.7683	1.8855	1.9757	2.0483	2.1090
∞	1.3396	1.6039	1.7659	1.8809	1.9694	2.0408	2.1005

TABLE 2

Factors for obtaining 100(1-β)% 1-sided exceedances for containing at least ℓ-t+1 out of ℓ additional observations at a third stage given a first stage sample of size n and a second stage sample of size m(=ℓ) for which at least m-k+1 out of m observations were in the prediction interval (Model B).

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40	1.8686	2.0889	2.2277	2.3275	2.4048	2.4675	2.5200
50	1.8683	2.0877	2.2258	2.3258	2.4031	2.4659	2.5186
60	1.8677	2.0862	2.2243	2.3239	2.4011	2.4640	2.5167
70	1.8671	2.0850	2.2227	2.3221	2.3993	2.4621	2.5148
80	1.8665	2.0839	2.2214	2.3205	2.3976	2.4603	2.5130
90	1.8660	2.0829	2.2202	2.3192	2.3962	2.4588	2.5115
∞	1.8596	2.0719	2.2061	2.3031	2.3734	2.4399	2.4915

k= 2, t= 6, β= .10

ℓ / n	20	30	40	50	60	70	80
2	.6716	.8383	.9945	1.0573	1.0935	1.1275	1.1430
4	.9955	1.2417	1.3774	1.4654	1.5273	1.5748	1.6117
6	1.1217	1.3807	1.5233	1.6276	1.6995	1.7543	1.7930
8	1.1863	1.4517	1.6064	1.7112	1.7835	1.8486	1.8972
10	1.2244	1.4934	1.6521	1.7606	1.8413	1.9046	1.9560
12	1.2490	1.5202	1.6813	1.7924	1.8754	1.9408	1.9943
14	1.2659	1.5384	1.7012	1.8140	1.8987	1.9657	2.0206
16	1.2781	1.5513	1.7154	1.8294	1.9153	1.9835	2.0395
18	1.2872	1.5609	1.7258	1.8408	1.9276	1.9966	2.0534
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40	1.3222	1.5948	1.7614	1.8792	1.9692	2.0414	2.1015
50	1.3268	1.5985	1.7649	1.8826	1.9723	2.0454	2.1058
60	1.3296	1.6005	1.7666	1.8843	1.9745	2.0472	2.1078
70	1.3315	1.6018	1.7675	1.8851	1.9753	2.0480	2.1086
80	1.3328	1.6026	1.7680	1.8855	1.9756	2.0483	2.1089
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Prediction intervals are extended to a third sampling stage involving the distribution of the t-th smallest value in a third stage sample which exceeds the k-th smallest value in the second sample. Procedures and tables are given for two situations. In the first situation the usual 2-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for this problem. In the second situation we know in advance that three stages will be necessary and the factors are given for the required procedure.

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